International Journal of Engineering Research & Management Technology

ISSN: 2348-4039

March-2016 Volume 3, Issue-2

Email: editor@ijermt.org www.ijermt.org

ROUGH BI-QUASI-IDEALS IN Γ-SEMIGROUPS

Dr. V.S.Subha

Assistant Professor, Department of Mathematics

Annamalai Unversity, Annamalainagar-608002, India

ABSTRACT:

This paper as further generalization of rough ideals. We introducethe notion of rough bi-quasi ideals of a Γ-semigroups as a generalization of rough bi-ideals of Γ-semigroup. We study the properties of rough bi-quasi ideal characterize the rough bi-quasi simple Γ-semigroups and regular Γ-semigroups using rough bi-quasi ideals.

INDEX TERMS: Rough bi-quasi-ideal, rough simple -semigroups, roughregular -semigroups

1. INTRODUCTION

Sen[16] introduced the notion of Γ-semigroupsas a generalization of semigroups. Sha[14,15] extended many classical notions of semigroup to Γ-semigroup.Many researchers studied ideals through Γ-semigroups such as Chinram et al.,[3] discussed bi-ideals in Γ-semigroups,Marapureddy Murali Krishna rao[8] introduced bi-quasi ideal in Γ-semigroups.

The notion of rough set was introduced by Pawlak in his papers[19-12]. Rough set theory is an extension of set theory, in which a sub set of a universe is described by a pair of ordinary sets called the lower and upper approximations. The lower approximation of a given set is the union of all the equivalence classes which have a non empty intersection with the set. It is well known that a partition induces an equivalence relation on a set and vice versa. Rough sets are a suitable mathematical model or vague concepts, i.e., concepts without sharp boundaries.

Some authors have studied the algebraic properties of rough sets. Bonikowski [2], Iwinski [5] and Pomykala[13] studied algebraic properties of rough sets.Biswas and Nanda[1] introduced the notion of rough subgroups. Kuroki[7] introduced the notion of rough ideals in semigroups. Jun[6], Chinram[4], Thillaigovindan et.al.,[19,20] introduced rough ideals in Γ-semigroups.V.S.Subha [17,18] introduced rough *k*-ideals and quasi-ideals in semirings.

In this paper we introduce the concept of rough bi-quasi-ideals and characterize rough bi-quasi simple Γ -semigroup and regular Γ-semigroup using rough bi-quasi-ideals.

2.PRELIMINARIES:

In this section we will recall some of the fundamental concepts and definitions which are necessary for the paper. Through out this paper M denotes aΓ -semigroup unless otherwise mentioned

A nonempty subset A of M is called

(i) a Γ *-semigroup* of M if $A\Gamma A \subseteq A$

(ii) a *quasi-ideal* of M if $A\Gamma M \cap M\Gamma A \subseteq A$

(iii) a *bi-ideal* of M if $A\Gamma A \subseteq A$ and $A\Gamma M\Gamma A \subseteq A$

(iv) an *interior ideal* of M if $A\Gamma A \subseteq A$ and $M\Gamma A\Gamma M \subseteq A$

(v) a *left(right)* of M if $M\Gamma A \subseteq A(A\Gamma M \subseteq A)$

(vi) an *ideal* of M if $M\Gamma A \subseteq A$ and $A\Gamma A \subseteq A$.

Definition 2.1: An element$a \in M$is said to be *regularelement* of$M$, if thereexists$x \in M$, $\alpha, \beta \in \Gamma$such that $a = a\alpha x\beta a$.If every element of M is regular element of M then M is said to be *regular*Γ-semigroup.

Let θ be a congruence relation on M, that is θ is an equivalence relation on M such that $(a,b) \in \theta \Rightarrow (a\gamma x, b\gamma x) \in \theta$ and $(x\gamma a, x\gamma b) \in \theta$ for all $a, x, b \in \Gamma$. If θ is a congruence relation on M, then for every $x \in M$, $[x]_\theta$ denotes the congruence class of x with respect to the relation θ. A congruence relation θ on M is called complete if $[a]_\theta \Gamma [b]_\theta = [a\Gamma b]_\theta$ for every $a, b \in M$.

Let A be a nonempty subset of M. Then the sets $\underline{\theta}(\mathrm{A}) = \{\mathrm{x} \in \mathrm{M}/[\mathrm{x}]_\theta \subseteq \mathrm{A}\}$ and $\overline{\theta}(\mathrm{A}) = \{\mathrm{x} \in \mathrm{M}/[\mathrm{x}]_\theta \cap \mathrm{A} \neq \phi\}$ are called the θ-lower and θ-upper approximations of A respectively. For a nonempty subset A of M,$\theta(\mathrm{A}) = (\overline{\theta}(\mathrm{A}), \underline{\theta}(\mathrm{A}))$ is called rough set with respect to θ if$\underline{\theta}(\mathrm{A}) \neq \overline{\theta}(\mathrm{A})$.

Theorem 2.2[19] *Let θ be a congruence relation on M. If A and B are non empty subsets of M. Thenthe following are true.*

i) $\underline{\theta}(A) \subseteq A \subseteq \overline{\theta}(A)$

ii) $\underline{\theta}(\emptyset) \subseteq \emptyset \subseteq \overline{\theta}(\emptyset)$

iii) $\underline{\theta}(M) = M = \overline{\theta}(M)$ and$\underline{\theta}(\Gamma) \subseteq \Gamma \subseteq \overline{\theta}(\Gamma)$

iv) $\overline{\theta}(A \cup B) = \overline{\theta}(A) \cup \overline{\theta}(B)$.

v) $\underline{\theta}(A \cap B) = \underline{\theta}(A) \cap \underline{\theta}(B)$.

vi) $\overline{\theta}(A)\Gamma\overline{\theta}(B) \subseteq \overline{\theta}(A\Gamma B)$.

vii) If θ is complete, then $\underline{\theta}(A)\Gamma\underline{\theta}(B) \subseteq \underline{\theta}(A\Gamma B)$.

Definition 2.3:[19] Letθbe a complete congruence relation on$M$. A non empty subset A of M is called a *θ-upper* (resp.*θ-lower*) *rough Γ-semigroup* of M if $\overline{\theta}(A)$ (resp.$\underline{\theta}(A)$) is a *Γ-semigroup*of M.

Definition 2.4:[19] Letθbe a complete congruence relation on$M$. A non empty subset A of M is called a *θ-upper* (resp.*θ-lower*)
rough bi-ideal of M, if $\overline{\theta}(A)$ (resp.$\underline{\theta}(A)$) is a *bi-ideal* of M.

Definition 2.5[19] Letθbe a complete congruence relation on$M$.A non empty subset A of M is called a *θ-upper* (resp.*θ-lower*) *rough quasi-ideal* of M if $\overline{\theta}(A)$ (resp.$\underline{\theta}(A)$) is a *quasi-ideal* of M.

Theorem 2.6[19] *Let θ be a congruence relation on M. If A is a left (resp. right) ideal of M and $\theta(A)$ is non empty then,*

(i) *A is a θ-upper rough left (resp. right) ideal of M.*

(ii) *A is a θ-lower rough left (resp. right) ideal of M.*

Theorem 2.7[19]*Let θ be a congruence relation on M. If A is a Γ-semigroup of M and $\underline{\theta}(A)$ non empty then,*

(i) *A is a θ-upper rough subsemigroup of M.*

(ii) *A is a θ-lower rough subsemigroup of M.*

Theorem 2.8[19]*Let θ be a congruence relation on M. If A is a bi-ideal of M and $\underline{\theta}(A)$is non empty then,*

(i) *A is a θ-upper rough bi- ideal of M.*

(ii) *A is a θ-lower rough left (resp. right) ideal of M.*

Theorem 2.9[19] *Let θ be a congruence relation on M and let A bea left (resp. right)ideal of M then,*

(i) *$\overline{\theta}(A)$ is both a bi-ideal and quasi-ideal of M.*

(ii) *If θ is complete and $\underline{\theta}(A) \neq \emptyset$ then $\underline{\theta}(A)$ is both a bi-ideal and quasi-ideal of M.*

Theorem 2.10[19]*Let θ be a congruence relation on M. If Q is a quasi-ideal of M and $\underline{\theta}(A)$ is non empty then,*

(i) *Q is a θ-upper rough quasi- ideal of M.*

(ii) *Q is a θ-lower rough quasi-ideal of M.*

Theorem 2.11[19]*Let θ be a congruence relation on M. If Q is a quasi-ideal of M and $\underline{\theta}(A)$ is non empty then,*

(i) *Q is a θ-upper rough bi- ideal of M.*

(ii) *Q is a θ-lower rough bi-ideal of M.*

3. Rough Bi-Quasi-Ideals in Γ-semigroups

In this section, as a generalization of rough ideals, we introduce the notion of rough left(right) bi-quasi-ideal and rough bi-quasi- ideal of Γ-semigroup. We study the properties of rough bi-quasi ideals of Γ -semigroup.

Definition 3.1.A nonempty subset L of M is said to be *left*(*resp.right*) *bi-quasi-ideal*of M if L is a Γ-subsemigroup of M and $M\Gamma L \cap L\Gamma M\Gamma L \subseteq L$ $(L\Gamma M \cap L\Gamma M\Gamma L \subseteq L)$

Definition 3.2.A nonempty subset L is said to be *bi-quasi-ideal* if it is both aleft bi-quasi ideal and right bi-quasi ideal of M .

Definition 3.3. M is called a *bi-quasi simple Γ- semigroup* if M has no bi-quasi-ideal other than M itself.

Definition 3.4. Let θ be a complete congruence relation on M. A nonemptysubset L of M is called θ -*upper* (resp.*lower*) *rough left*(resp.*right*) *bi-quasi-ideal* of M if $\overline{\theta}(L)$(resp.$\underline{\theta}(L)$)is a left(resp.right) *bi-quasi ideal* of M.

Theorem 3.5.*Let θ be a congruence relation on M. If L is a left(resp.right) bi-quasi-ideal of M . Then ,*

(i) *L is a θ-upper rough left(resp.right) bi-quasi-ideal of M.*

(ii) *If θ is complete if $\underline{\theta}(L)$is nonempty, then L is a θ-lower rough left(resp.right) bi-quasi-idealof M.*

Proof: Let L be aleft(resp.right) bi-quasi-ideal of M. Then L is an ideal of M.

(i) By Theorem 2.6(i) $\overline{\theta}(L)$ is a left(resp.right) ideal of M, and by Theorem 2.9(i) $\overline{\theta}(L)$ is a bi-ideal of M. Then $M\Gamma\overline{\theta}(L) \subseteq \overline{\theta}(L)$, and $\overline{\theta}(L)\Gamma M\Gamma\overline{\theta}(L) \subseteq \overline{\theta}(L)$.

Therefore $M\Gamma\overline{\theta}(L) \cap \overline{\theta}(L)\Gamma M\Gamma\overline{\theta}(L) \subseteq M\Gamma(\overline{\theta}(L) \cap \overline{\theta}(L)) \subseteq M\Gamma\overline{\theta}(L) \subseteq \overline{\theta}(L)$.

Thus $\overline{\theta}(L)$ is a bi-quasi-ideal of M

(ii) By Theorem 2.6(ii) $\underline{\theta}(L)$is a left(resp.right) ideal of M, and by Theorem 2.9(ii) $\underline{\theta}(L)$is a bi-ideal of M. Then $M\Gamma\underline{\theta}(L) \subseteq \underline{\theta}(L)$, and $\underline{\theta}(L)\Gamma M\Gamma\underline{\theta}(L) \subseteq \underline{\theta}(L)$.

Therefore $M\Gamma\underline{\theta}(L) \cap \underline{\theta}(L)\Gamma M\Gamma\underline{\theta}(L) \subseteq M\Gamma(\underline{\theta}(L) \cap \underline{\theta}(L)) \subseteq M\Gamma\underline{\theta}(L) \subseteq \underline{\theta}(L)$.

Thus $\underline{\theta}(L)$ is a bi-quasi-ideal of M

Theorem 3.6.*Let θ be a complete congruence relation on M. Every leftideal of M, is a*

(i) *θ-upper rough left bi-quasi-ideal of M.*

(ii) *If the lower approximationof the left ideal is non empty, then it is θ-lower rough left bi-quasi-ideal of M.*

Proof: Let L is a left ideal of M. Then by Theorem 2.6(i)$\overline{\theta}(L)$is a left ideal of $M$and by Theorem 2.9(i) $\overline{\theta}(L)$is a bi-ideal of M

$M\Gamma\overline{\theta}(L) \cap \overline{\theta}(L)\Gamma M\Gamma\overline{\theta}(L) \subseteq M\Gamma(\overline{\theta}(L) \cap \overline{\theta}(L))$,by Theorem 2.9(i)

$\subseteq M\Gamma\overline{\theta}(L) \subseteq \overline{\theta}(L)$

Hence $\overline{\theta}(L)$ is a left bi-quasi-ideal of M.

(ii) Similarly we prove $\underline{\theta}(L)$ is a left bi-quasi-ideal of M

Theorem 3.7.*Let θ be a complete congruence relation on M. Every left ideal of M, is a*

(i) *θ-upper rough right bi-quasi-ideal of M.*

(ii) *If the lower approximationof the left ideal is non empty, then it is θ-lower rough right bi-quasi-ideal of M.*

Proof: Let L is a left ideal of M.

(i) By Theorem 2.6(i)$\overline{\theta}(L)$is a left ideal of M.

$$\overline{\theta}(L)\Gamma M \cap \overline{\theta}(L)\Gamma M\Gamma\overline{\theta}(L) \subseteq (\overline{\theta}(L)\Gamma M) \cap (\overline{\theta}(L)\Gamma M)\Gamma\overline{\theta}(L)$$

$$\subseteq \overline{\theta}(L)\Gamma M\Gamma\overline{\theta}(L)$$

$$\subseteq \overline{\theta}(L)\Gamma\overline{\theta}(L) \subseteq \overline{\theta}(L).$$

Thus$\overline{\theta}(L)$ is a right bi-quasi-ideal of M.

(ii) Similarly we prove $\underline{\theta}(L)$ is a right bi-quasi-ideal of M.

Corollary 3.8. *Let θ be a complete congruence relation on M. Every left idealof M, is a*

(i) *θ-upper rough bi-quasi-ideal of M.*

(ii) *If the lower approximationof the left ideal is non empty, then it is θ-lower rough bi-quasi-ideal of M.*

Proof: Let L be a left ideal of M. By Theorem 3.6 and Theorem 3.7 L is θ-upperand θ-lowerrough bi-quasi-ideal of M.

Theorem 3.9.*Let θ be a complete congruence relation on M. Every quasi-ideal of M, is a*

(i) *θ-upper rough bi-quasi-ideal of M.*

(ii) *If the lower approximation of the quasi-ideal is non empty, then it is θ-lower rough bi-quasi-ideal of M.*

Proof: Let Q is a quasi-ideal of M.

(i) By Theorem 2.11(i) $\overline{\theta}(Q)$ is a bi-ideal of M, then $\overline{\theta}(Q)\Gamma M\Gamma\overline{\theta}(Q)\subseteq\overline{\theta}(Q)$. We have $\overline{\theta}(Q)\Gamma M\Gamma\overline{\theta}(Q)\subseteq\overline{\theta}(Q)\Gamma M$. Since $M\Gamma\overline{\theta}(Q)\subseteq\overline{\theta}(Q)\subseteq$ M.

Therefore

$M\Gamma\overline{\theta}(Q)\cap\overline{\theta}(Q)\Gamma M\Gamma\overline{\theta}(Q)\subseteq M\Gamma\overline{\theta}(Q)\cap\overline{\theta}(Q)\Gamma M\subseteq\overline{\theta}(Q)$.

Hence $\overline{\theta}(Q)$ is a left bi-quasi-ideal of M.

Similarly we prove $\overline{\theta}(Q)$ is a right bi-quasi-ideal of M.

(ii) Similar to (i)

Theorem 3.10.*Let θ be a complete congruence relation on M. Every bi-ideal of M, is a*

(i) *θ-upper rough bi-quasi-ideal of M.*

(ii) *If the lower approximation of the bi-ideal is non empty, then it is θ-lower rough bi-quasi-ideal of M.*

Proof: Let B is a bi-ideal of M.

(i) By Theorem 2.9(i) $\overline{\theta}(B)$ is a bi-ideal of M, then $\overline{\theta}(B)\Gamma \mathrm{M}\Gamma\overline{\theta}(B)\subseteq\overline{\theta}(B)$.

Therefore

$M\Gamma\overline{\theta}(B)\cap\overline{\theta}(B)\Gamma M\Gamma\overline{\theta}(B)\subseteq M\Gamma\left(\overline{\theta}(B)\cap\overline{\theta}(B)\right)\Gamma M\Gamma\overline{\theta}(B)$

$\subseteq M\Gamma\left(\overline{\theta}(B)\Gamma M\Gamma\overline{\theta}(B)\right)$

$\subseteq M\Gamma\overline{\theta}(B)$

$\subseteq\overline{\theta}(B)$,since $\mathrm{M}\Gamma\overline{\theta}(B)\subseteq\overline{\theta}(B)$.

Hence $\overline{\theta}(B)$ is a left bi-quasi-ideal of M. Similarly we prove $\overline{\theta}(B)$is a right bi-quasi-ideal of M.

(ii) Similar to (i)

Theorem 3.11.*Let θ be a complete congruence relation on M, the intersection of a family of bi-quasi-ideal of $M$is a rough bi-quasi-ideal of M.*

Proof : Considr a subsemigroup $L=\bigcap_{i\in 1}L_i$, of M, where L_i is a bi-quasi-ideal of M.

By Theorem 2.7 , $\overline{\theta}(L)$ and $\underline{\theta}(L)$ are sub semigroup of M..

$M\Gamma\left(\cap\overline{\theta}(L_i)\right)\cap\left(\cap\overline{\theta}(L_i)\right)\Gamma M\Gamma\left(\cap\overline{\theta}(L_i)\right)\subseteq M\Gamma\overline{\theta}(L_i)\cap\overline{\theta}(L_i)\Gamma M\Gamma\overline{\theta}(L_i)\subseteq\overline{\theta}(L_i)$.

This implies that $M\Gamma\left(\cap\overline{\theta}(L_i)\right)\cap\left(\cap\overline{\theta}(L_i)\right)\Gamma M\Gamma(\cap\overline{\theta}(L_i))\subseteq\cap\overline{\theta}(L_i)$.

Hence $\bigcap_{i\in 1}\overline{\theta}(L_i)$ is a left bi-quasi-ideal of M.

By Theorem 3.7 $\bigcap_{i\in 1}\overline{\theta}(L_i)$ is also a right bi-quasi-ideal of M.

Thus $\bigcap_{i\in 1}\overline{\theta}(L_i)$ is a bi-quasi-ideal of M.

Similarly we have prove $\bigcap_{i\in 1}\underline{\theta}(L_i)$is a bi-quasi-ideal of M.

Therefore $\theta(L)=(\overline{\theta}(L),\underline{\theta}(L))$ is a rough bi-quasi-ideal of M.

Corollary 3.12.*Let θ be a complete congruence relation on M. If L is a left ideal and R is a right ideal of M then,*

i) *$\overline{\theta}(L\cap R)$is a bi-quasi-ideal of M.*

ii) $\underline{\theta}(L\cap R)$is a bi-quasi-ideal of M.

Proof : By Theorem 2.2 and 3.11(i) $\overline{\theta}(L \cap R)$is a bi-quasi-ideal of M.
By Theorem 2.2 and 3.11(ii) $\underline{\theta}(L \cap R)$is a bi-quasi-ideal of M.

Theorem 3.13.*Let θ be a complete congruence relation on M. For a Γ-semigroup M, the following are eqivalent*

i) *M is bi-quasi simple Γ-semigroup.*
ii) *$\theta(M)\Gamma a = \theta(M)$, for all $a \in M$*
iii) $(a) = \theta(M)$ for all $a \in M$ where (a) is the smallest bi-quasi-ideal generated by a.

Proof : Let M be a Γ-semigroup.
(i) $\Rightarrow$ (ii) Suppose that $\overline{\theta}(M)$ is a bi-quasi simple Γ-semigroup, $a \in \overline{\theta}(M)$ and $\overline{\theta}(L) = \overline{\theta}(M)\Gamma a$. Then $\overline{\theta}(L)$ isa left ideal. By Corollary 3.8 $\overline{\theta}(L)$ is a bi-quasi-ideal of M. Hence $\overline{\theta}(M)\Gamma a = \overline{\theta}(M)$ for all $a \in \overline{\theta}(M)$.
By Theorem 2.2(iii) $\underline{\theta}(M) = \underline{\theta}(M)\Gamma a$.
Therefore $\theta(M)\Gamma a = \theta(M)$.
(ii) $\Rightarrow$ (iii)Suppose that $\overline{\theta}(M)\Gamma a = \overline{\theta}(M)$ for all $a \in \overline{\theta}(M)$ and (a) is the smallest bi-quasi-ideal of $\overline{\theta}(M)$ containing a. Then $\overline{\theta}(M)\Gamma a \subseteq (a) \Rightarrow \overline{\theta}(M) \subseteq \overline{\theta}(M)$.
Hence$\overline{\theta}(M) = (a)$.By Theorem 2.2(iii) $\underline{\theta}(M) = (a)$.
Therefore $\theta(M) = (a)$.
(iii) $\Rightarrow$ (i) Suppose $(a)$is the smallest bi-quasi-ideal generated by a, $\overline{\theta}(M) = (a)$ for all $a \in \overline{\theta}(M)$ and $\overline{\theta}(A)$ is the bi-quasi simple Γ-semigroup of M and $a \in \overline{\theta}(A)$.Then $(a) \subseteq \overline{\theta}(A) \subseteq \overline{\theta}(M)$. Then $\overline{\theta}(A) = \overline{\theta}(M)$ and by Theorem 2.2(iii) $\overline{\theta}(M) = M = \underline{\theta}(M)$. Hence M is a bi-quasi simple Γ-semigroup.

Theorem 3.14.*Let θ be a complete congruence relation on M. Then M is a bi-quasi simple Γ-semigroup if and only if $\theta(M)\Gamma a \cap a\Gamma\theta(M)\Gamma a = \theta(M)$, for all $a \in \theta(M)$.*

Proof: Suppose $\overline{\theta}(M)$ is a bi-quasi simple Γ-semigroup and $a \in \overline{\theta}(M)$. By Corollary 3.12 $\overline{\theta}(M)\Gamma a \cap a\Gamma\overline{\theta}(M)\Gamma a$ is a bi-quasi-ideal of M. Therefore

$\overline{\theta}(M)\Gamma a \cap a\Gamma\overline{\theta}(M)\Gamma a = \overline{\theta}(M)$ for all $a \in \overline{\theta}(M)$, since $\overline{\theta}(M)$ is a -quasi simple Γ-semigroup.

Conversely assume that $\overline{\theta}(M)\Gamma a \cap a\Gamma\overline{\theta}(M)\Gamma a = \overline{\theta}(M)$, for all $a \in \overline{\theta}(M)$. Let $\overline{\theta}(T)$ be a bi-quasi-ideal of M and $a \in \overline{\theta}(T)$.

$$\overline{\theta}(M) = \overline{\theta}(M)\Gamma a \cap a\Gamma\overline{\theta}(M)\Gamma a \subseteq \overline{\theta}(M)\Gamma\overline{\theta}(T) \cap \overline{\theta}(T)\Gamma\overline{\theta}(M)\Gamma\overline{\theta}(T) \subseteq \overline{\theta}(T) \subseteq \overline{\theta}(M)$$

Therefore $\overline{\theta}(M) = \overline{\theta}(T)$. Hence $\overline{\theta}(M)$ is is a bi-quasi simple Γ-semigroup.

The similar argument is true for $\underline{\theta}(M)$.

Theorem 3.15.*Let θ be a complete congruence relation on M. If $M\Gamma a = M$ for all $a \in M$ then every left bi-quasi-ideal of M is a rough quasi-ideal of M.*

Proof: Suppose $M\Gamma a = M$ *for all* $a \in M$and L is left bi-quasi-ideal of M.

By Theorem 3.3 $\overline{\theta}(L)$ is a left bi-quasi-ideal of M. Then

$M\Gamma\overline{\theta}(L) \cap \overline{\theta}(L)\Gamma M\Gamma\overline{\theta}(L) \subseteq \overline{\theta}(L)$. Let $a \in \overline{\theta}(L)$. Thus $M\Gamma a \subseteq M\Gamma\overline{\theta}(L)$ and

$M \subseteq M\Gamma\overline{\theta}(L) \subseteq M$. This implies that $M\Gamma\overline{\theta}(L) \cap \overline{\theta}(L)\Gamma M \subseteq \overline{\theta}(L)$.

Therefore $\overline{\theta}(L)$ is a quasi-ideal of M.

Similarly we prove $\underline{\theta}(L)$ is a quasi ideal of M.

Corollary 3.16.*Let θ be a complete congruence relation on M. If $M\Gamma a = M$ for all $a \in M$ then every bi-quasi-ideal of M is a rough quasi-ideal of M.*

Theorem 3.17. *Let θ be a complete congruence relation on M and M is a regular Γ-semigroup. Then every quasi-ideal of M is an rough ideal of M.*

Proof: Let Q be a quasi-ideal of M. By Theorem 2.10 $\overline{\theta}(Q)$ is a quasi-ideal of M. Then $\overline{\theta}(Q)\Gamma M \cap M\Gamma\overline{\theta}(Q) \subseteq \overline{\theta}(Q)$. Thus $\overline{\theta}(Q) \subseteq M \Rightarrow M\Gamma\overline{\theta}(Q) = \overline{\theta}(Q)\Gamma M$. Therefore
$M\Gamma\overline{\theta}(Q) = M\Gamma\overline{\theta}(Q) \cap \overline{\theta}(Q)\Gamma M \subseteq \overline{\theta}(Q)$. Since $\overline{\theta}(Q)$ is a quasi-ideal of M.
Similarly $\overline{\theta}(Q)\Gamma M \subseteq \overline{\theta}(Q)$. Thus $\overline{\theta}(Q)$ is an ideal of M.
In similar way we have to prove $\underline{\theta}(Q)$ is an ideal of M.

Theorem 3.18. *Let θ be a complete congruence relation on M. Then M is a regular Γ-semigroup if and only if $\theta(A)\Gamma\theta(B) = \theta(A) \cap \theta(B)$, for any right ideal A and left ideal B of M.*

Proof : Let A be the right ideal and B be the left ideal of M. By Theorem 2.6(i) $\overline{\theta}(A)$ be the right ideal and $\overline{\theta}(B)$ be the left ideal of M. Then $\overline{\theta}(A)\Gamma\overline{\theta}(B) \subseteq M\Gamma\overline{\theta}(B) \subseteq \overline{\theta}(B)$ and $\overline{\theta}(A)\Gamma\overline{\theta}(B) \subseteq \overline{\theta}(A)\Gamma M \subseteq \overline{\theta}(A)$.
This implies that $\overline{\theta}(A)\Gamma\overline{\theta}(B) \subseteq \overline{\theta}(A) \cap \overline{\theta}(B)$. ____________________(1)

Then by Corollary 3.12 $\overline{\theta}(A) \cap \overline{\theta}(B)$ is a bi-quasi-ideal of M. Then

$$\overline{\theta}(A) \cap \overline{\theta}(B) = M\Gamma\left(\overline{\theta}(A) \cap \overline{\theta}(B)\right) \cap \left(\overline{\theta}(A) \cap \overline{\theta}(B)\right)\Gamma M\Gamma\left(\overline{\theta}(A) \cap \overline{\theta}(B)\right)$$

$$\subseteq \left(\overline{\theta}(A) \cap \overline{\theta}(B)\right)\Gamma M\Gamma\left(\overline{\theta}(A) \cap \overline{\theta}(B)\right)$$

$$\subseteq \overline{\theta}(A)\Gamma M\Gamma\overline{\theta}(B) \subseteq \overline{\theta}(A)\Gamma\overline{\theta}(B)$$

$\overline{\theta}(A) \cap \overline{\theta}(B) \subseteq \overline{\theta}(A)\Gamma\overline{\theta}(B)$ ____________________(2).

By (1) and (2) $\overline{\theta}(A) \cap \overline{\theta}(B) = \overline{\theta}(A)\Gamma\overline{\theta}(B)$

Theorem 3.19. *Let θ be a complete congruence relation on M and let M is a regular Γ-semigroup. Then every left bi-quasi-ideal of M is a rough ideal of M.*

Proof : Let M be a regular Γ-semigroup and L be a left bi-quasi-ideal of M. By Theorem 3..3 $\overline{\theta}(L)$ is a left bi-quasi-ideal of M. Then $M\Gamma\overline{\theta}(L) \cap \overline{\theta}(L)\Gamma M\Gamma\overline{\theta}(L) \subseteq \overline{\theta}(L)$. We know that $\overline{\theta}(L)\Gamma M$ and $M\Gamma\overline{\theta}(L)$ are right ideal and left ideal of M. By Theorem 3.19

$$\left(\overline{\theta}(L)\Gamma M\right)\Gamma\left(M\Gamma\overline{\theta}(L)\right) = \left(\overline{\theta}(L)\Gamma M\right) \cap \left(M\Gamma\overline{\theta}(L)\right).$$ Therefore

$$\left(\overline{\theta}(L)\Gamma M\right) \cap \left(M\Gamma\overline{\theta}(L)\right) = \left(\overline{\theta}(L)\Gamma M\right)\Gamma\left(M\Gamma\overline{\theta}(L)\right) \subseteq M\Gamma\overline{\theta}(L)$$ and

$\left(\overline{\theta}(L)\Gamma M\right) \cap \left(M\Gamma\overline{\theta}(L)\right) = \left(\overline{\theta}(L)\Gamma M\right)\Gamma\left(M\Gamma\overline{\theta}(L)\right) \subseteq \overline{\theta}(L)\Gamma M\Gamma\overline{\theta}(L)$. Hence

$\left(\overline{\theta}(L)\Gamma M\right) \cap \left(M\Gamma\overline{\theta}(L)\right) \subseteq M\Gamma\overline{\theta}(L) \cap \overline{\theta}(L)\Gamma M\Gamma\overline{\theta}(L) \subseteq \overline{\theta}(L)$. Thus $\overline{\theta}(L)$ is a quasi-ideal of M. By Theorem 3.18 $\overline{\theta}(L)$ is an ideal of M.

Similarly we prove $\underline{\theta}(L)$ is an ideal of M.

Therefore $\theta(L) = \left(\underline{\theta}(L), \overline{\theta}(L)\right)$ is a rough ideal of M.

Corollary 3.20. *Let θ be a complete congruence relation on M and let M is a regular Γ-semigroup. Then every bi-quasi-ideal of M is a rough ideal of M.*

Theorem 3.21. *Let θ be a complete congruence relation on M and let M is a regular Γ-semigroup if and only if $\theta(B) = M\Gamma\theta(B) \cap \theta(B)\Gamma M\Gamma\theta(B) \subseteq \theta(B)$ for every left bi-quasi-ideal B of M.*

Proof : Suppose B is a left bi-quasi-ideal of M. By Theorem 3..3 $\overline{\theta}(B)$ is a left bi-quasi-ideal of M and let $x \in \overline{\theta}(B)$. We have $M\Gamma\overline{\theta}(B) \cap \overline{\theta}(B)\Gamma M\Gamma\overline{\theta}(B) \subseteq \overline{\theta}(B)$, since $\overline{\theta}(B)$ is a left bi-quasi-ideal of M. *As* M is regular there exists $y \in M, \alpha, \beta \in \Gamma$ such that $x = x\alpha y\beta x$. Then $x \in M\Gamma\overline{\theta}(B)$ and $\overline{\theta}(B)\Gamma M\Gamma\overline{\theta}(B)$. Therefore $\overline{\theta}(B) \subseteq \overline{\theta}(B)\Gamma M\Gamma\overline{\theta}(B) \cap M\Gamma\overline{\theta}(B) = \overline{\theta}(B)$.

Conversely Suppose that $\overline{\theta}(B) = \overline{\theta}(B)\Gamma M\Gamma\overline{\theta}(B) \cap M\Gamma\overline{\theta}(B)$, for any left bi-quasi-ideal $\overline{\theta}(B)$ of M.Let $\overline{\theta}(R)$ and $\overline{\theta}(L)$ be right and left ideal of M respectively. Then by Corollary 3.12 $\overline{\theta}(R) \cap \overline{\theta}(L)$ is a bi-quasi-ideal of M. Then

$\overline{\theta}(R) \cap \overline{\theta}(L) = M\Gamma\left(\overline{\theta}(R) \cap \overline{\theta}(L)\right) \cap \left(\overline{\theta}(R) \cap \overline{\theta}(L)\right)\Gamma M\Gamma\left(\overline{\theta}(R) \cap \overline{\theta}(L)\right)$

$\subseteq \left(\overline{\theta}(R) \cap \overline{\theta}(L)\right)\Gamma M\,\Gamma\left(\overline{\theta}(R) \cap \overline{\theta}(L)\right)$

$\subseteq \overline{\theta}(R)\Gamma M\Gamma\overline{\theta}(L)$

$\subseteq \overline{\theta}(R)\Gamma\overline{\theta}(L)$

we have $\overline{\theta}(R)\Gamma\overline{\theta}(L) \subseteq \overline{\theta}(L)$ and $\overline{\theta}(R)\Gamma\overline{\theta}(L) \subseteq \overline{\theta}(R)$.

Therefore $\overline{\theta}(R)\Gamma\overline{\theta}(L) \subseteq \overline{\theta}(R) \cap \overline{\theta}(L)$. Hence $\overline{\theta}(R)\Gamma\overline{\theta}(L) = \overline{\theta}(R) \cap \overline{\theta}(L)$. By Theorem 3.19, M is a regular Γ-semigroup.

Similar argument is true for $\underline{\theta}(B)$.

Corollary 3.22: *Let θ be a complete congruence relation on M and let M is a regular Γ-semigroup if and only if $\theta(B) = M\Gamma\theta(B) \cap \theta(B)\Gamma M\Gamma\theta(B) \subseteq \theta(B)$ for every bi-quasi-ideal B of M.*

5. CONCLUSION

The rough set theory is regarded as a generalization of the classical set theory. A key notion in rough set is an equivalence relation. An equivalence is sometime difficult to be obtained in reward problems due to vagueness and incompleteness of human knowledge.In this paper we introduce the concept of rough bi-quasi ideals and characterize rough bi-quasi simple Γ -

semigroup and regular Γ -semigroup.We plan to study rough fuzzy bi-quasi-ideals in Γ -semigroup.

REFERENCES

1. R. Biswas and S. Nanda, *Rough groups and rough subgroups*, Bulietin polish Academy Science Mathematics 42(1994) 251-254.
2. Z.Bonikowaski, *Algebraic structures of rough sets*,in: W.P.Ziarko(Ed), Rough Sets, Fuzzy Sets And Knowledge Discovery, Springer-Verlag, Berlin, 1995,pp 242-247.
3. R. Chinram and C. Jirojkul, *on bi-ideals in Γ-semigroups*, Journal of science and Technology 29(1)(2007) 231-234.
4. R.Chinram, *Rough Prime ideals and rough Fuzzy Prime ideals in Γ-semigroups*, Communication of the Koren Mathematical Society,24(3)(2009)341-351.
5. T. Iwinski, *Algebraic approach to rough sets*, Bull. Polish. Acad.Sci. and Math. 35(1987), 673-683.
6. Y.B. Jun, *Roughness of Gamma-subsemigroup and ideals in Γ-semigroup*, Bulletin of Korean Mathematical Society, 40(3) (2003), 531-536.
7. N. Kuroki, *Rough Ideals in semigroups*, Information Sciences, 100(1997), 130-163.
8. Marapureddy Murali Krishna rao, *Bi-quasi ideals and fuzzy bi-quasi ideals of Γ -semigroups*,Bulletin of the international Mathamatical virtual Institute, Vol 7(2017),231-242.
9. Z. Pawlak, *Rough sets*, International Journal of Information Computer Science, 11 (1982)341-356.dgji
10. Z. Pawlak, *Rough sets and fuzzy sets*, Fuzzy sets and systems, 17(1)(1985)99-102.
11. Z. Pawlak and A.Showron, *Rough sets:some extensions*, Information Sciences, 177(1) (2007)28-40.
12. Z. Pawlak, *Rough sets-Theoretical aspects of reasoning about data*, Kuluwer Academic publishers,Dordrecht(1991).
13. J. Pomykala, J.A. Pomykala, The stone algebra of roughsets, Bull. Polish. Acad. Sci. Math. 36(1998)
14. N.K. Saha, *On Γ -semigroup II*, Bulletin of Calcutta Mathematical society, 79(6) (1987) 331-335.
15. N.K. Saha, *On Γ -semigroup III*, Bulletin of Calcutta Mathematical society, 80(1) (1988) 1-12.
16. M.K. Sen, *On -semigroup*, Proceedings of International Symposium on Algebra and Applications,Decker Publications, New York(1982) 301-368.
17. V.S.Subha, *Rough k-ideals in Semirings*, International Journal of Research Publication & Seminar, Volume 05(01) March, 2014, 117-126.
18. V.S.Subha, *Rough Quasi-ideals in Regular Semirings*, International Journal of Research Publication & Seminar, Volume 06(01), February, 2015, 116-123.
19. N. Thillaigovindan, V. Chinnadurai and V.S. Subha, *Rough ideals in Γ - semigroups*, InternationalJournal of Algebra.
20. N. Thillaigovindan, V.S. Subha, *Rough prime bi-ideals in Γ - semigroups*,The journal of Fuzzy mathematics, Vol 23, No 1, 189-197, 2015.